

Math 62 : Word problems for Final Exam

Math 72 : 3.2 Word problems

Math 70 B 2.4 & 2.5 Word Problems

(We are skipping forward to the first sections that can be done without a graphing calculator.)

2.4 Applications with Percent**Objectives**

- 1) Write percent as decimal.
- 2) Write decimal as percent.
- 3) Solve basic percent problems.

2.5 Problem Solving**Objectives**

Solve problems using

- 1) direct translation
- 2) consecutive integers, consecutive even integers, or consecutive odd integers
- 3) geometry formulas
 - a. perimeter
 - b. area of a rectangle
 - c. angles of a triangle
 - d. complementary or supplementary angles
- 4) money, including
 - a. fixed and variable costs
 - b. break-even analysis of cost and revenue
- 5) percent increase or percent decrease
- 6) uniform motion ($D=RT$)

Examples & Practice Problems

Define a variable, write an algebraic equation using that variable, and solve the equation to answer the question. If the question includes units, always include them in your answer.

- 1) Six times some number added to 8 amounts to 4 added to the product of 4 and the number.
- 2) The sum of three consecutive even integers is 228. Find the integers.
- 3) The perimeter of a cross section of a "two by four" piece of lumber is 10 inches. The length is 2 inches longer than the width. Find the actual dimensions of the cross section of a two-by-four.
- 4) The length of a rectangle is two feet more than the width. The area is 131.25 square feet. Find the dimensions of the rectangle.
- 5) Find the measures of the angles of a triangle if the measure of the first angle is three times the measure of the second angle and the third angle is 25° more than the second angle.

- 6) Angle A and angle B are complementary angles and angle A is 14° more than three times angle B. Find the measures of angle A and angle B.
- 7) Angle A and angle B are supplementary angles and angle A is 35° less than four times angle B. Find the measures of angle A and angle B.
- 8) Fine Line Trucks rents an 18-ft truck for \$42 plus 35 cents per mile. Judy needs a truck for one day to deliver a shipment. How far can she drive and stay within a budget of \$70?
- 9) The cost C to produce x number of tennis rackets is $C = 170 + 13x$. The tennis rackets are sold wholesale for \$18 each, so revenue R is given by $R = 18x$. Find how many tennis rackets the manufacturer needs to produce and sell to break even.
- 10) Find 13% of 5000.
- 11) The population of a town increased by 200%, to 100 people. Find the original population of the town.
- 12) A diamond ring sold for \$2875.50 including tax. If the tax rate where the diamond was purchased is 6.5%, find the price of the ring before the tax was added. (Round to the nearest cent, if necessary.)
- 13) During a stock market "correction", Eva's retirement funds lost 8% of their value. After the crash, Eva had \$180,000 in her retirement account. How much was in the account before the crash?
- 14) Rafaela ran for 20 minutes and then walked for 10 minutes. Her running rate was 250 feet per minute faster than her walking rate. If she ran and walked a total of 15,500 feet, how fast did she run?
- 15) Antonio drove for 2 hours in a snowstorm and then for 5 hours more in clear weather. He drove half as fast through the snow as he did in the clear weather. If he drove 240 more miles in clear weather than he did in the snow, how fast did he drive through the snow?
- 16) Emory rode their bicycle at a rate of 15 mph and then slowed to 10 mph. They rode 30 minutes longer at 15 mph than they did at 10 mph. If they traveled a total of 25 miles, how long did they ride at the faster rate? [Caution: Be careful the units match!]

Using Dimensions to Remember Formulas and Units

For any geometric formula, the degree of the formula = dimension of the concept = exponent on units

ONE DIMENSION: degree 1 formula, units with exponent 1 (invisible 1)

TWO DIMENSIONS: degree 2 formula, units with exponent 2 or the word "square".

THREE DIMENSIONS: degree 3 formula, units with exponent 3 or the word "cubic".

Concept	Degree of formula	Using formula to find units	Dimension = Degree
Length	$L = L^1$, degree 1	$L = 3 \text{ feet} = \text{feet}^1$	1
Perimeter of a Rectangle	$P = 2L + 2W$ Formula has two terms. Degree of $2L = 2L^1$ is 1. Degree of $2W = 2LW^1$ is 1. Maximum of 1 and 1 is 1. Degree of $2L + 2W$ is 1.	$L = 3 \text{ feet} = 3 \text{ feet}^1$ $W = 2 \text{ feet} = 2 \text{ feet}^1$ $P = 2(3 \text{ feet}) + 2(2 \text{ feet})$ $= 6 \text{ feet} + 4 \text{ feet}$ $= 10 \text{ feet}$ [combine like terms]	1
Circumference of a Circle	$C = 2\pi R$ Formula has one term. 2 is a constant. π is a constant. Degree of $2\pi R^1$ is 1.	$R = 3 \text{ feet} = 3 \text{ feet}^1$ $C = 2\pi R$ $= 2\pi(3 \text{ feet})$ $= 6\pi \text{ feet}$ $\approx 18.84 \text{ feet}$	1
Area of a Rectangle	$A = LW$ Formula has one term. Degree of $LW = L^1W^1$ is $1+1=2$.	$L = 3 \text{ feet} = 3 \text{ feet}^1$ $W = 2 \text{ feet} = 2 \text{ feet}^1$ $A = 3 \text{ feet}^1 \cdot 2 \text{ feet}^1$ $= 6 \text{ feet}^2 = 6 \text{ square feet.}$ [exponent laws]	2
Volume of a Rectangular Solid	$V = LWH$ Formula has one term. Degree of $LWH = L^1W^1H^1$ is $1+1+1=3$.	$L = 3 \text{ feet} = 3 \text{ feet}^1$ $W = 2 \text{ feet} = 2 \text{ feet}^1$ $H = 4 \text{ feet} = 4 \text{ feet}^1$ $V = 3 \text{ feet} \cdot 2 \text{ feet} \cdot 4 \text{ feet}$ $= 24 \text{ feet}^3 = 24 \text{ cubic feet.}$ [exponent laws]	3
Volume of a Sphere	$V = \frac{4}{3}\pi R^3$ Formula has one term. $\frac{4}{3}$ is a constant. π is a constant. Degree of $\frac{4}{3}\pi R^3$ is 3.	$R = 3 \text{ feet} = 3 \text{ feet}^1$ $V = \frac{4}{3}\pi R^3$ $= \frac{4}{3}\pi(3 \text{ feet})^3$ $= \frac{4}{3}\pi \cdot 3^3 \cdot \text{feet}^3$ [exponent laws] $= 36\pi \text{ feet}^3$ $= 36\pi \text{ cubic feet}$ $\approx 113.04 \text{ feet}^3$	3

BACKGROUND YOU NEED:

Determine the degree of a polynomial:

- 1) Identify the terms, separated by add or subtract
- 2) For each term, use only the variable letters, not the constant numbers.
- 3) For each term, add up exponents on the variables
- 4) The degree of the polynomial is the maximum of the degrees of the terms.

Example: Find the degree of $-3xy^3z + 4x^2y$.

There are two terms. The degree of $-3x^1y^3z^1$ is $1+3+1=5$

The degree of $4x^2y^1$ is $2+1=3$

The maximum of 5 and 3 is 5.

The degree of $-3x^1y^3z^1 + 4x^2y$ is 5.

Combine like terms:

- 1) Check: are the terms like terms, having the same variables with the same exponents on those variables? Unlike terms cannot be combined.
- 2) Add the coefficients, while keeping the variables and exponents unchanged.

Example: Simplify $2xy - 5xy + 8xz$.

$2xy$ and $-5xy$ are like terms, but $8xz$ is not.

The coefficients are 2 and -5.

The answer is $-3xy + 8xz$

Exponent Law: Multiply like bases means add exponents

When two factors with like bases which are variables are multiplied, we can add the exponents.

Example: Multiply $2xyz^2 \cdot (-3xy^3z)$.

The factors are 2, x, y, z^2 , -3, x, y^3 , and z.

The like bases are x with x, y with y^3 , and z^2 with z.

Multiply: $2 \cdot (-3) \cdot x \cdot x \cdot y \cdot y^3 \cdot z^2 \cdot z$

$= -6 \cdot x^{1+1} \cdot y^{1+3} \cdot z^{2+1}$

The answer is $-6x^2y^4z^3$

Exponent Law: Exponent Outside Parentheses is applied to all bases inside parentheses

Example: Simplify $4(-2x)^3$

Notice that 4 is outside the parentheses. The exponent 3 does not apply to it.

Notice that the negative is inside the parentheses. The exponent 3 applies to $-2, x$.

$= 4 \cdot (-2)^3 \cdot x^3$

$= 4 \cdot (-8) \cdot x^3$

The answer is $-32x^3$.

PHRASES MEANING ADDITION

Phrase	Translation
"the sum of 3 and a number"	$3 + x$
"the total of a number and 50"	$x + 50$
"8 more than a number"	$x + 8$
"a number increased by 22"	$x + 22$
"a number plus 10"	$x + 10$
"one number added to another"	$x + y$

PHRASES MEANING SUBTRACTION

Phrase	Translation
"4 subtracted from a number"	$x - 4$
"the difference of 4 and a number"	$4 - x$
"4 decreased by a number"	$4 - x$
"4 less than a number"	$x - 4$
"4 fewer than a number"	$x - 4$
"4 minus a number"	$4 - x$
"4 reduced by a number"	$4 - x$
"4 less a number"	$4 - x$

PHRASES MEANING MULTIPLICATION

Phrase	Translation
"a number multiplied by 5"	$5 \cdot x$ or simply $5x$
(We usually write variable factors last, so we write $5x$ rather than $x5$.)	
"the product of 3 and a number"	$3x$
"one number times another"	xy
" $\frac{2}{3}$ of a number"	$\frac{2}{3}x$
<i>Special multiplication phrases</i>	
"twice a number"	$2x$
"double a number," "a number doubled"	$2x$
"triple a number," "a number tripled"	$3x$

PHRASES MEANING DIVISION

Phrase	Translation
"a number divided by 8"	$x \div 8$ or $\frac{x}{8}$
"the quotient of a number and 8"	$x \div 8$ or $\frac{x}{8}$
"the ratio of a number to 8"	$x \div 8$ or $\frac{x}{8}$
"a number divided into 8"	$x \overline{)8}$ or $8 \div x$
<i>Special division phrase</i>	
"the reciprocal of a number"	$\frac{1}{x}$

KEY VERBS AND VERB PHRASES MEANING "EQUALS"

Phrase	Translation
"equals" or "is equal to"	$=$
"is" or "are" or "was"	$=$
"gives" or "gives us"	$=$
"gives a result of" or "results in"	$=$
"leaves us" or "leaves" or "left"	$=$
"is the same as"	$=$

PHRASES MEANING POWERS OR ROOTS

Phrase	Translation
"the square of a number"	x^2
"the cube of a number"	x^3
"a number raised to the fourth power"	x^4
"the sixth power of a number"	x^6
"The square root of a number"	$\sqrt{x}$

Math 45: Useful Math Terminology

Operation: add, subtract, multiply, divide, exponent, radical, absolute value, etc.

Order of Operations: Rules for which part of a calculation to do first, and then what order to proceed

Variable: A letter used to represent any or a specific number, often unknown

Expression: A combination of variables and numbers with no equal sign, though operations are permitted

Substitute: Replace a variable or an expression by a variable or expression which is known to be equal

Evaluate: Give a number answer as a result, often by substituting given values for a variable then performing any operations

Terms: Parts of an expression that are separated by addition (rewrite subtraction as addition)

Distribute: Multiply a single term by all terms within parentheses that follow or precede it

Factors: Parts of a term that are separated by multiplication (rewrite division as multiplication)

Factor: Find the prime factors (numbers or polynomials) which multiply to give the original expression

Reduce: Put a fraction in lowest terms. A reduced fraction has no common factors, decimals or fractions within fractions

Constant: An expression which is a number without variables

Coefficient: A number, especially if multiplied times a variable or product of variables or quotient of variables

Equation: Two expressions separated by an equal sign

Inequality: (a) One of the four symbols $<$, $>$, $\leq$, $\geq$
(b) Two expressions separated by an inequality symbol

Solve: Find all solutions, meaning the values for a variable which make an equation or inequality true. Solution may be a single value, list, set, interval, or a graph.

Isolate: Perform operations to both sides of an equation or inequality so the isolated item is alone on one side

Set equal to zero: (a) Set an expression equal to zero: write the expression, an equal sign, and a zero when finding the x-intercepts or zeros of the expression viewed as a function.

(b) Set an equation equal to zero: Rearrange terms by adding or subtracting the same term from both sides of the equation until one side is zero.

Rational: A number or expression which is or can be written as a fraction of integers or a fraction of polynomials

Simplify: Final form for an answer. Reduce fractions, distribute, combine like terms, evaluate operations. If simplifying a rational expression, factor and divide out common factors, then leave final answer factored.

Interval: A set of all values between two given endpoints. Notation specifies if endpoints are included.

Ordered Pair or Point: A pair of numbers in parentheses, separated by a comma, denoting the x- and y-coordinates on a rectangular coordinate graph

Geometry Formulas

DEFINITIONS

The **perimeter** is the sum of the lengths of all the sides of a figure.

The **area** is the amount of space enclosed by a two-dimensional figure measured in units squared.

The **surface area** of a solid is the sum of the areas of the surfaces of a three-dimensional figure.

The **volume** is the amount of space occupied by a three-dimensional figure measured in units cubed.

The **radius** r of a circle is the line segment that extends from the center of the circle to any point on the circle.

The **diameter** of a circle is any line segment that extends from one point on the circle through the center to a second point on the circle. The diameter is two times the length of the radius, $d = 2r$.

In circles, we use the term **circumference** to mean the perimeter.

Plane Figures

Formulas

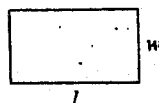
Square



$$\text{Area: } A = s^2$$

$$\text{Perimeter: } P = 4s$$

Rectangle



$$\text{Area: } A = lw$$

$$\text{Perimeter: } P = 2l + 2w$$

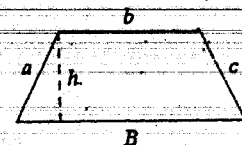
Triangle



$$\text{Area: } A = \frac{1}{2}bh$$

$$\text{Perimeter: } P = a + b + c$$

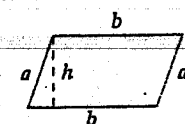
Trapezoid



$$\text{Area: } A = \frac{1}{2}h(b + B)$$

$$\text{Perimeter: } P = a + b + c + B$$

Parallelogram



$$\text{Area: } A = bh$$

$$\text{Perimeter: } P = 2a + 2b$$

Circle



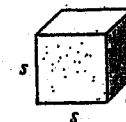
$$\text{Area: } A = \pi r^2$$

$$\text{Circumference: } C = 2\pi r = \pi d$$

Solids

Formulas

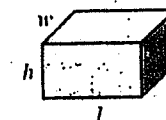
Cube



$$\text{Volume: } V = s^3$$

$$\text{Surface Area: } S = 6s^2$$

Rectangular Solid



$$\text{Volume: } V = lwh$$

$$\text{Surface Area: } S = 2lw + 2lh + 2wh$$

Sphere



$$\text{Volume: } V = \frac{4}{3}\pi r^3$$

$$\text{Surface Area: } S = 4\pi r^2$$

Right Circular Cylinder



$$\text{Volume: } V = \pi r^2 h$$

$$\text{Surface Area: } S = 2\pi r^2 + 2\pi rh$$

Cone



$$\text{Volume: } V = \frac{1}{3}\pi r^2 h$$

① Six times some number added to 8 amounts to

translate $6 \cdot x + 8 =$

4 added to the product of 4 and the number.

translate $4 + 4 \cdot x$

Resulting equation: $6x + 8 = 4 + 4x$

collect variables $\frac{-4x}{-4x}$

$$2x + 8 = 4$$

isolate variable:

$$\frac{-8}{-8} \quad \frac{-8}{-8}$$

$$2x = -4$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$\boxed{x = -2}$$

② The sum of 3 consecutive even integers is 228.

1st 2nd 3rd

$$\boxed{} + \boxed{} + \boxed{} = 228$$

consecutive: in a row

even: evenly divisible by 2

integer: no fraction or decimal part

ex:

50, 52, 54

or 66, 68, 70

$$\boxed{x} + \boxed{x+2} + \boxed{x+4} = 228$$

+2
+4

M70

$$x + x + 2 + x + 4 = 228$$

$$3x + 6 = 228$$

$$\begin{array}{r} -6 \\ \hline \end{array} \quad \begin{array}{r} -6 \\ \hline \end{array}$$

$$3x = 222$$

$$\frac{3x}{3} = \frac{222}{3}$$

$$x = 74$$

$$x + 2 = 74 + 2 = 76$$

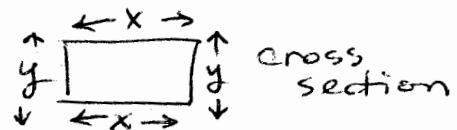
$$x + 4 = 74 + 4 = 78$$

74, 76, 78

combine like terms

isolate x

- ③ The perimeter of a cross-section of a "two by four" piece of lumber is 10 inches



perimeter: add the lengths of sides, going around the edge:

$$x + y + x + y = 10$$

$$\frac{2x}{2} + \frac{2y}{2} = \frac{10}{2} \quad \text{or} \quad \underline{\underline{(x) + y = 5}}$$

The length is 2" longer than width. $x = (y + 2)$
Find actual dimensions.

substitute for x :
in 1st (perimeter)
equation

$$(y + 2) + y = 5$$

$$\begin{array}{r} 2y + 2 = 5 \\ -2 \quad -2 \\ \hline 2y = 3 \end{array}$$

M20

$$\frac{2y}{2} = \frac{3}{2}$$

$$y = \frac{3}{2} \text{ in or } 1.5 \text{ in}$$

$$x = y + 2 \Rightarrow x = \frac{3}{2} + 2 \text{ or } 1.5 + 2$$

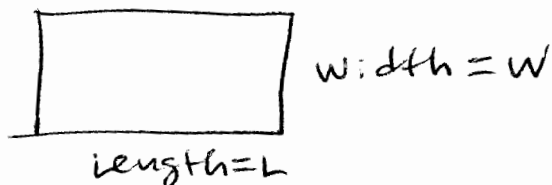
$$= \frac{3}{2} + \frac{4}{2} = 3.5 \text{ in}$$

$$= \frac{7}{2} \text{ in}$$

dimensions: $1.5 \text{ in} \times 3.5 \text{ in}$

or $\frac{3}{2} \text{ in} \times \frac{7}{2} \text{ in}$

- ④ The length of a rectangle is 2 feet more than the width. The area is 131.25 square feet. Find the dimensions of the rectangle



$$\text{Length} = 2 + \text{Width} \Rightarrow L = (2 + W)$$

$$\text{Area} = L \cdot W \Rightarrow 131.25 = (L)W$$

$$\text{Substitute for L: } 131.25 = (2 + W) \cdot W$$

$$\text{distribute } 131.25 = 2W + W^2$$

↑

uh-oh!

exponent 2 is

- called "quadratic"

- is not linear

- cannot be solved by isolating the variable

STOP!

We won't be able to solve this question until

a) we can use a GC

or b) we learn quadratics (chapter 11)

⑤ Find the measures of the angles of a triangle if

3 angles, add to 180°

$$\boxed{\text{1st}} + \boxed{\text{2nd}} + \boxed{\text{3rd}} = 180$$

... The measure of the first angle is 3 times the measure of the second

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{(1st)} & = & 3 \cdot \text{(2nd)} \end{array}$$

... and the third angle is 25° more than the second angle

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{(3rd)} & = & 25 + \text{(2nd)} \end{array}$$

Notice that both (1st) and (3rd) are equal to expressions containing (2nd).

$\Rightarrow$ They can be written in terms of 2nd.

$\Rightarrow$ This means that (2nd) is an excellent choice for a variable

Let $x = \text{2nd angle} \Rightarrow \text{(2nd)} = x$

$$\text{(1st)} = 3x$$

$$\text{(3rd)} = 25 + x$$

Substitute these into the boxes we wrote at first:

$$\boxed{3x}_{\text{(1st)}} + \boxed{x}_{\text{(2nd)}} + \boxed{25+x}_{\text{(3rd)}} = 180$$

$$3x + x + 25 + x = 180$$

$$5x + 25 = 180$$

$$\begin{array}{r} -25 \\ \hline \end{array} \quad \begin{array}{r} -25 \\ \hline \end{array}$$

$$5x = 155$$

remove boxes

combine like terms on LHS

isolate x

M20

$$\frac{5x}{5} = \frac{155}{5}$$

isolate x

$$x = 31^\circ$$

$$3x = 3(31) = 93^\circ$$

$$25 + x = 31 + 25 = 56^\circ$$

$$\text{check: } 31 + 93 + 56 = 180^\circ \checkmark$$

$$\text{answer: } \boxed{31^\circ, 93^\circ, 56^\circ}$$

⑥ Angles A and B are complementary angles

two angles $\Rightarrow$
two boxes

$$\boxed{} + \boxed{} = 90^\circ$$

$A = (1st) \quad B = (2nd)$

... and angle A is 14° more than three times angle B.

$$A = 14 + 3 \cdot B \Rightarrow A = 14 + 3B$$

$$\left. \begin{array}{l} (A) + B = 90 \\ A = (14 + 3B) \end{array} \right\} \text{substitute}$$

$$(14 + 3B) + B = 90$$

$$\begin{array}{r} 14 + 4B = 90 \\ -14 \quad -14 \\ \hline \end{array} \quad \text{combine like terms}$$

$$\frac{4B}{4} = \frac{76}{4}$$

isolate x (B is this case)

$$\boxed{B = 19^\circ}$$

$$A = 14 + 3B \quad \text{or} \quad A + B = 90$$

$$A = 14 + 3(19)$$

$$A + 19 = 90$$

$$\Rightarrow \boxed{A = 71^\circ}$$

⑦ Angle A and angle B are supplementary angles...

$$\boxed{A} + \boxed{B} = 180^\circ$$

A (1st) B (2nd)

...and angle A is 35° less than 4 times angle B.

$$A = 4 \cdot B - 35$$

less than

means

- subtract
- in the opposite order of their appearance in the sentence

$$\left. \begin{array}{l} (A) + B = 180 \\ A = (4B - 35) \end{array} \right\} \text{substitute}$$

$$(4B - 35) + B = 180$$

$$5B - 35 = 180$$

$$\begin{array}{r} +35 \\ 5B - 35 = 180 \\ \hline 5B = 215 \end{array}$$

$$\begin{array}{r} 5B = 215 \\ \hline 5 \quad 5 \\ B = 43 \end{array}$$

$$\boxed{B = 43^\circ}$$

combine like terms

isolate variable B

$$A = 4B - 35$$

$$A = 4(43) - 35$$

or $A + B = 180$

$$A + 43 = 180$$

$$\boxed{A = 137^\circ}$$

⑧ Fine-line trucks rents an 18-foot truck ...
 ↓
 irrelevant.

... for \$42 plus 35 cents per mile.

pay only once
 cost → 42 + .35 (# miles)
 add convert to dollars multiply by # of miles

... Judy needs a truck for one day to deliver a shipment.
How far can she drive and stay within a budget of \$70?

- distance, measured in miles
- This is the question! let $x = \# \text{ miles}$.

↓
 less than
 or equal to
 \$70.

$$\text{cost} \rightarrow 42 + .35x \leq 70.$$

or just $42 + .35x = 70$ for maximum number of miles

$$\begin{array}{r} -42 \\ \hline \end{array}$$

$$\begin{array}{r} -42 \\ \hline \end{array}$$

$$\begin{array}{r} .35x = 28 \\ \hline .35 \quad .35 \end{array}$$

isolate x

$$x = \boxed{80 \text{ miles}}$$

* always include units

⑨ The cost C to produce x tennis rackets is $C = 170 + 13x$.

↓
 we are manufacturers!
 Imagine that!

... The tennis rackets are sold wholesale for \$18 each

↓
 reword "for \$18 per racket"
 ↓
 mult by #
 of rackets.

... So revenue is given by $R = 18x$

↳ cash into the cash register — no bills paid yet!

$$R = 18x$$

$R = 18x$
 $\uparrow$ multiply by $x = \#$ rackets, same as $(\#8)$ "per".

... Find how many tennis rackets to produce to break-even.

costs = revenues
 ↳ ↳
 Pay bills enough money

After the break-even point our business makes money.

Costs = Revenues

$$170413x = 18x$$

$$\begin{array}{r} -13x \\ \hline \end{array} \quad \begin{array}{r} -13x \\ \hline \end{array}$$

collect variables on same side

$$\frac{170}{5} = \frac{5 \times}{5}$$

$$\frac{170}{5} = \frac{5 \times}{5}$$

isolate x

$$34 = 4x$$

Sell 34 rackets to break even

* always include units!

(10) Find 13% of 5000. base

find 13% of 5000. $\swarrow$ base
 $\searrow$ of means multiply
 a portion

$.13 = \frac{13}{100}$, as a fraction or decimal

$$a = (.13)(5000)$$

$$a = 650$$

Basic percent
 $a = p \cdot b$

- ① the population increased by 200%, to 100 people.
Find the original population of the town.

Key question: 200% of what?

either 200% of unknown starting population $\Rightarrow 2.00x$

OR 200% of ending population 100 $\Rightarrow 2(100) = 200$?!?

In percent increase OR percent decrease problems,
it is always $(\text{percent}) \times \left(\frac{\text{earlier base value}}{\text{value}} \right)$

So $2x$ is correct and 200 is incorrect.

Percent increase equation

$$b + p \cdot b = \text{future}$$

← our question says
"increase"

Percent decrease equation

$$b - p \cdot b = \text{future}$$

b = earlier
base value,
unknown x .

$$x + 2.00x = 100$$

$$3x = 100$$

$$x = \frac{100}{3}$$

$$x \approx 33\frac{1}{3} \approx \text{about } \boxed{33 \text{ people}}$$

$\frac{1}{3}$ person doesn't
make sense
So round to
give meaningful
answer.

(12) A diamond sold for \$2875.50 including tax.
↓

The money listed
is
price + tax

$\therefore$ If the tax rate is 6.5%, find price before tax.

$\downarrow$
earlier
base b
is unknown

↓
.065 as decimal

Tax is added to the price $\Rightarrow$ percent increase
 $b + p \cdot b = 2875.50$.

$$b + .065b = 2875.50$$

$$1b + .065b = 2875.50$$

$$(1 + .065)b = 2875.50$$

$$1.065b = 2875.50$$

$$\frac{1.065b}{1.065} = \frac{2875.5}{1.065}$$

$$b = \$2700$$

Combine like terms

$$b + b = 2b$$

$$.1b + .1b = .2b$$

$$b + .1b = (1 + .1)b = 1.1b$$

isolate variable

easier examples

(13) During a stock market correction, Eva's

The value of stocks
"crashed" or
went down, decreased. $\Rightarrow$ percent decrease

... retirement funds lost 8% of their value

$\downarrow$ $\downarrow$ $\downarrow$
 .08 as decimal multiply original value, earlier in time

... After the crash, Eva had \$180,000 in her account

$\downarrow$
 \$180,000 is the ending
 or final amount

... How much was in the account before the crash?

$\rightarrow$ find original
 base amount = b .

percent decrease equation

$$b - p \cdot b = \text{future}$$

$$b - .08b = 180000$$

$$(1 - .08)b = 180000$$

$$.92b = 180000$$

$$\frac{.92b}{.92} = \frac{180000}{.92}$$

$$b = 195652.1739$$

$$\boxed{\$195,652.17}$$

round to nearest
 hundredth (cent)
 to make sense for
 money

combine like term
 $\begin{cases} 2b - b = b \\ 1b - .1b = (1 - .1)b \\ \quad = .9b \end{cases}$
 easier examples

(14) Rafael ran 20 min then walked 10 min. His running
 ↓ ↓
 time also time

... rate was 250 ft per min faster than his walking rate.

↓

Keyword
"rate"

↓

add
250 ft/min to

↗

his walking
rate

... If he ran and walked a total of 15500 feet, how
 ↓ ↓
 add together distance

... fast did he run?

fixed rate for running. $\Rightarrow x = \text{rate for running}$

"Distance", "Rate", "Time" $\Rightarrow$ key words mean $D = R \cdot T$

	D	R	T
run	(?)	x	20 min
walk	(?)	250 ft/min <u>slower</u> than x	10 min

total: 15E00

To make a table, spread out the formula, make one line for running and one line for walking.

Note: we could set this up with x = rate for walking
and $x + 250$ = rate for running,
and do an extra step at the end.

"Distance = rate times time" is true for walking and it's true for running. Mult R.T and put the result in D.

$$D = R \cdot T$$

20 x	x	20
10(x-250)	x-250	10

total 15500

↑↑

"total distance" means "add the distances"

$$20x + 10(x-250) = 15500$$

distribute $20x + 10x - 2500 = 15500$

combine $30x - 2500 = 15500$

isolate $\quad \quad \quad +2500 \quad \quad +2500$

$$\frac{30x}{30} = \frac{18000}{30}$$

$$x = \boxed{600 \text{ ft/min}}$$

Realistic? Let's convert to mph:

$$600 \frac{\text{ft}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 6.8 \text{ miles per hour.}$$

Math 70

$10\times$ (is 240 miles more than) $2\times$

$\uparrow$ bigger

$\uparrow$ smaller

$2x + 240 = 10x$

↑
add to smaller expression

↑
larger expression

to make equal to

$$\begin{array}{r} 2x + 240 = 10x \\ -2x \qquad \qquad -2x \\ \hline \end{array}$$

collect x 's same side

$$\frac{240}{8} = \frac{8 \times}{8}$$

isolate x

$$30 = x$$

$x = 30$ mph in snow

(16) Emory rode their bicycle at a rate of 15 mph, then
↓
key word

... slowed to 10 mph. They rode 30 min longer at 15 mph

... than they did at 10 mph. If they traveled a total of 25 mile

time add 30 min to other time
↓
add ... → distances

... how long did they ride at the faster rate?
 ↓
 time keyword ↳ x = faster time

... Caution: Do units match??

Distance, Rate, Time $\Rightarrow D = R \cdot T$

$$D = R \cdot T$$

	15 mph	x
	10 mph	$x = 30 \text{ min}$

total 25 miles

miles per hour means $\frac{\text{miles}}{\text{hrs}}$

and implies time is in hours.

longer = bigger #

minutes!

shorter = smaller #

subtract

$$30 \text{ min} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{30}{60} = \frac{1}{2} \text{ hr} = .5 \text{ hr}$$

$$D = R \cdot T$$

$15x$	15 mph	x
$10(x+5)$	10 mph	$x+5$
Total: 25		

Math 70

add the distances to get the total:

$$15x + 10(x + 0.5) = 25$$

$$15x + 10x + 5 = 25$$

distribute

$$25x + 5 = 25$$

combine

$$\begin{array}{r} +5 \\ \hline \end{array} \quad \begin{array}{r} +5 \\ \hline \end{array}$$

isolate x

$$\frac{25x}{25} = \frac{20}{25}$$

$$\boxed{x = 1.2 \text{ hr}}$$

Does this make sense?

$$1.2 \text{ hr} \left(\frac{60 \text{ min}}{1 \text{ hr}} \right) = \boxed{72 \text{ min}}$$

yes, that's reasonable.

(11)

1.6.37

Define variables

The occupations of biomedical engineers, skin care specialists, and physician assistants are among the

10 with the largest growth from 2008 to 2018. [Translate this sentence #1] The number of physician assistant jobs will grow 2 thousand less than 3 times the number of biomedical engineer jobs. [The number of skin care specialist jobs will grow 15 thousand more than half the number of biomedical engineer jobs.] [sentence #2] If the total growth of these three jobs is predicted to be 58 thousand, find the predicted growth of each job. ↑ translate #3

The predicted growth of biomedical engineers is thousand. (Simplify your answer.)

The predicted growth of skin care specialists is thousand. (Simplify your answer.)

The predicted growth of physician assistants is thousand. (Simplify your answer.)

growth # Biomedical engineers = B
growth # Skin care specialists = S
growth # physician assistants = P

} 3 unknowns ⇒ need 3 equations

sentence #1

PA will grow 2000 less than 3 times BE
P = $3B - 2000$
↓ subtract reverse order

$$\underline{P = 3B - 2000}$$

an equation! hurray! 😊

sentence #2

SCS will grow 15 thousand more than half the BME.

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & & & & \\ S & = & 15000 & + & \frac{1}{2}B & & \end{array}$$

another equation! 😊

sentence #3

Total is predicted to be 58 thousand
↓
add up all 3

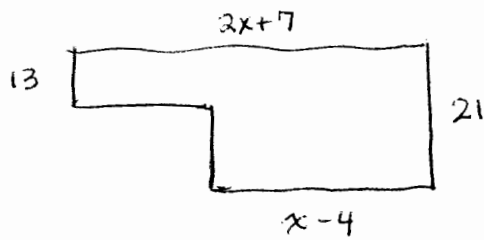
$$\underline{B + S + P = 58000}$$

another equation!

😊

Extras

- ⑫ a) Write an expression for the perimeter of this floor plan, and simplify result completely.

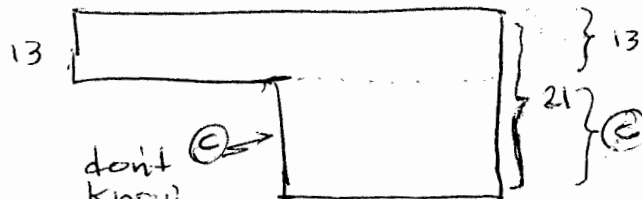


Notice! Two sides have not been labelled. ☹️

Perimeter = sum of lengths of sides

We assume floor plans have right angles, so this makes the three vertical lines parallel and the three horizontal lines parallel.

Method 1:



don't know

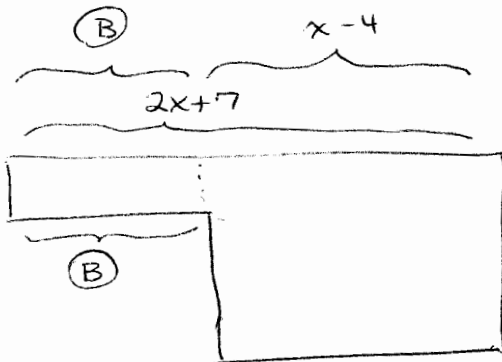
but can't

call it x , because x is already used in this problem to mean something else.

Solve for C

$$C = 21 - 13$$

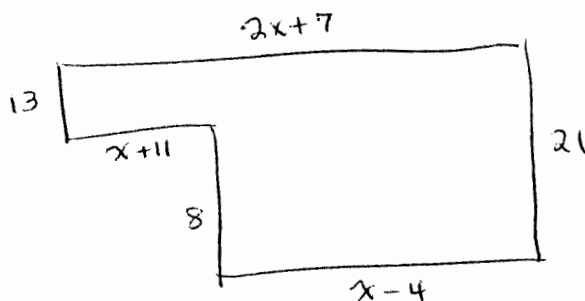
$$C = 8$$



← If we mentally slide all the horizontal lines to the top, we see that they add up

$$B + (x - 4) = (2x + 7)$$

$$\text{or } B + x - 4 = 2x + 7$$



Solve for B :

$$B = 2x + 7 - x + 4$$

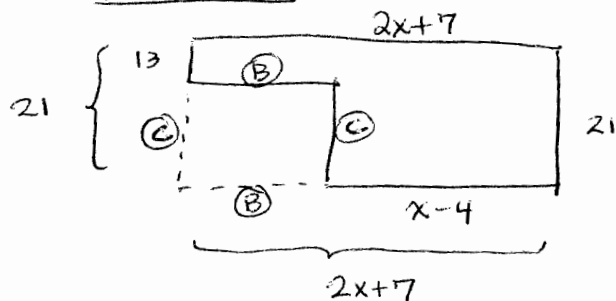
$$B = x + 11$$

Now we know all the sides.

Perimeter = add up all the sides

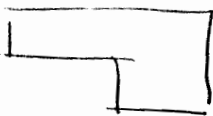
$$\begin{aligned}
 P &= (2x+7) + 21 + (x-4) + 8 + (x+11) + 13 \\
 &= 2x + x + x + 7 + 21 - 4 + 8 + 11 + 13 \\
 &= \boxed{4x + 56}
 \end{aligned}$$

Method 2:



Mentally slide the side (C) to the left. When added to 13, we get 21 from the opposite side.

Mentally slide the side (B) to the bottom. When added to $(x-4)$, we get $(2x+7)$ from the opposite side.

The perimeter of  is equivalent to

the perimeter of  !

$$P = 2(2x+7) + 2(21)$$

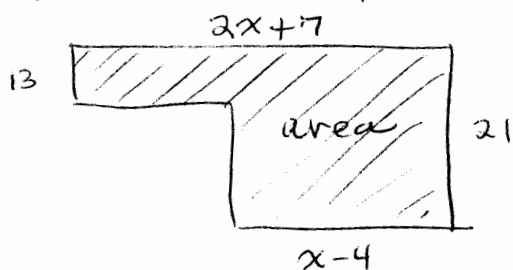
$$P = 4x + 14 + 42$$

$$\boxed{P = 4x + 56}$$

$$P = 2L + 2W$$

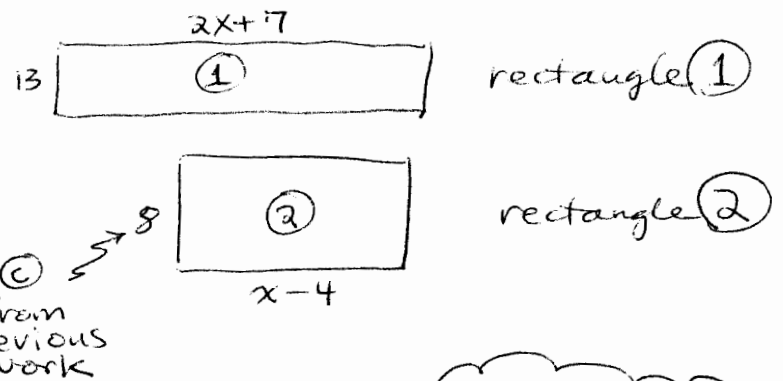
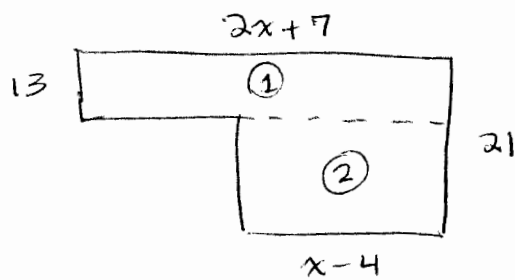
perimeter of rectangle

b) Write an expression for the area of the same figure



Area = amount of space inside figure

Method 1: Divide into two regions using a horizontal line



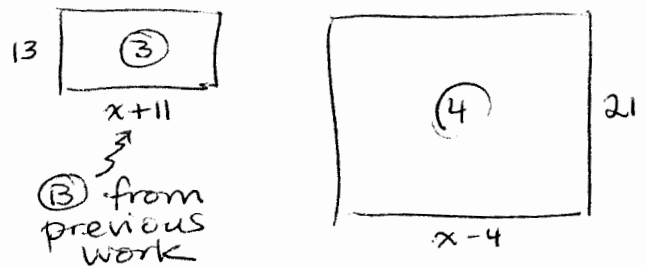
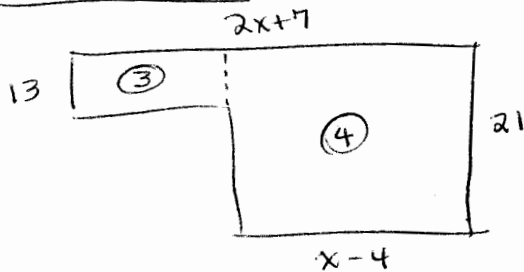
Total area = area of rectangle (1) + area of rectangle (2)

$A = L \cdot W$
area of a rectangle

$$\begin{aligned} \text{area} &= 13(2x+7) + 8(x-4) \\ &= 26x + 91 + 8x - 32 \end{aligned}$$

$$\boxed{A = 34x + 59}$$

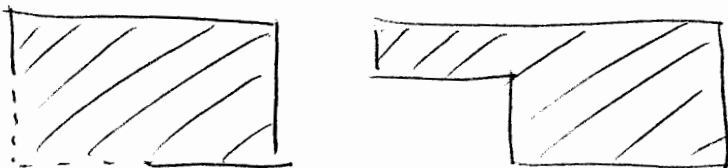
Method 2: Divide into two regions using a vertical line



$$\begin{aligned} \text{area} &= 13(x+11) + 21(x-4) \\ &= 13x + 143 + 21x - 84 \end{aligned}$$

$$\boxed{A = 34x + 59}$$

Question: Would we get the correct area by drawing a single rectangle as for the Perimeter Method 2?



No — the space inside (area) would be larger.

Math 70 1.6

c) If the perimeter is 80 ft, find x , A , and all sides.

$$P = 4x + 56 = 80$$

ignore P , solve for x .

$$4x + 56 = 80$$

$$4x = 24$$

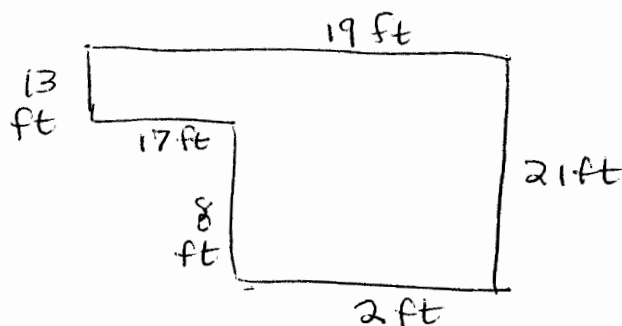
$$\boxed{x = 6 \text{ ft}}$$

$$A = 34x + 59$$

Substitute $x = 6$.

$$A = 34(6) + 59$$

$$\boxed{A = 263 \text{ ft}^2}$$



$$2x + 7$$

Subst $x = 6$

$$2(6) + 7$$

$$= 12 + 7$$

$$= 19 \text{ ft}$$

$$x - 4$$

Subst $x = 6$

$$6 - 4$$

$$= 2 \text{ ft}$$

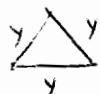
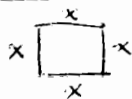
$$x + 11$$

Subst $x = 6$.

$$6 + 11$$

$$= 17 \text{ ft}$$

- 13) Two frames are needed with the same perimeter, one the shape of a square and one an equilateral Δ . Each side of the triangle is 6" longer than each side of the square. Find the dimensions of each frame. sq. 18", Δ . 24"



$$y = x + 6$$

$$4x = 3(x + 6)$$

$$x = 18''$$

$$y = 24''$$

- 14) The number of deaths caused by tornados decreased 59.2% from the 1950's to the 1990's. There were 579 deaths from tornados in the 1990's. How many deaths from tornados were there in 1950's. round to nearest whole

1419

$$\begin{array}{c} \swarrow 1950 \quad \nwarrow 1990 \\ x - .592x = 579 \end{array}$$

$$x = \frac{579}{1 - .592}$$

$$1419.1 \approx$$

$\boxed{1419 \text{ tornados in 1990's}}$

(15)

1.6.25

add a %

In 2006, the population of the country was 32.6 million. This represented an increase in population of 4.5% since 2001. What was the population of the country in 2001? Round to the nearest hundredth of a million.

In 2001, the population of the country was million
(Round to the nearest hundredth.)

$$\begin{array}{c} 2001 \\ \text{pop} \end{array} + \% \text{ of } \begin{array}{c} 2001 \\ \text{pop} \end{array} = \begin{array}{c} 2006 \\ \text{pop} \end{array}$$

$$= 32.6 \text{ million}$$

$$x + .045x$$

$$1x + 0.045x = 32.6 \text{ million} \quad \text{combine like terms}$$

$$1.045x = 32.6 \text{ million} \quad \text{isolate } x$$

$$x = \frac{32.6}{1.045}$$

$$x = 31.19617225$$

↑
hundredth of a million

$$x = \boxed{32.20 \text{ million people}}$$

$$1 \text{ million} = 1,000,000$$

$$32.20 \text{ million} = \boxed{32,200,000 \text{ people}}$$

A summary of the three types of percent problems is on the front of GC-9.

If you need a complete refresher, the Math 45 lesson on these percent problems is at the end of this packet.

Basic Percent $a = p \cdot b$

a = new amount

p = percentage, written as a decimal

b = base amount

Can also be done by proportion

$$\frac{p}{100} = \frac{a}{b}$$

a = new amount

p = percentage, written as percent

b = base amount

Percent Increase and Percent Decrease

$$b + pb = a \quad \text{increase}$$

$$b - pb = a \quad \text{decrease}$$

↑ ↑
must be the same, b = original base
earliest (in time)

p = percent written as decimal

a = new amount
later (in time)

Percent Increase problem types

- 1) sales tax
- 2) pay raise
- 3) investment gain
- 4) mark up
(wholesale to retail price)
- 5) population increase
- 6) inflation
- 7) tips
(restaurant servers)
- 8) "percent increase"
in wording

Percent Decrease problem types

- 1) pay cut
- 2) investment loss
- 3) discount or sale price
(retail price lowered)
- 4) population decrease
- 5) "percent decrease"
in wording

Detailed examples in the Math 45 lecture 2.6
at the end of this lecture

Math 45 SSM 2/e 2.6 problems with Percent - Day 2

Objectives

- Day 2 {
- 1) Apply the basic percent equation
 - 2) Apply the percent increase equation
 - 3) Apply the percent decrease equation.

Recall: Percent equation $a = pb$ means "a is p percent of b."

Percent increase means two steps in a single new formula

1. Find p percent of b.
2. Add result to b.

$$\text{new amount} = b + p \cdot b$$

$$\text{OR } \text{new amt} = (1+p)b$$

* CAUTION b must be the old amount, or the amount that was at an earlier time.

Must not confuse b and new amount or you'll get the wrong answer

Types of Problems which are percent increase:

- 1) Sales Tax
- 2) Pay Raise
- 3) Investment gain
- 4) Markup (wholesale to retail)
- 5) Population increase
- 6) Inflation
- 7) Tips
- 8) "Percent Increase"

Examples

- ① The sales tax rate is 5.75%. The total cost of a used Honda Civic, including sales tax, is \$8460. Find the cost before tax.

$$\begin{cases} \text{new amount} = 8460 \\ p = .0575 \\ b = \text{unknown} \end{cases}$$

Identify values

$$\text{new amount} = b + pb$$

$$8460 = b + .0575b$$

$$8460 = 1.0575b$$

$$\frac{8460}{1.0575} = b$$

$$b = \$8000$$

Write formula: $\frac{\text{Sales tax is}}{\text{Percent Increase}}$
Substitute

Combine like terms: $b = 1b$

CAUTION: COMMON MISTAKE:

You cannot avoid the algebra by doing only a basic percent;

$$a = p \cdot b$$

$$a = (.0575)(8460)$$

$$a = 486.45$$

$$8460 - 486.45 = \$7973.55$$

← **WRONG ANSWER** This is too small, since it's subtracting 5.75% of 8460, not 5.75% of 8000.

M45 SSM 2le 2.6 Day 2 cont p.2

- ② A college bookstore marks up 35% over publisher's wholesale price. A textbook sells for \$120. How much did it cost the bookstore?

Mark-up $\Rightarrow$ Percent increase

$$\text{new} = b + pb$$

$$\begin{cases} \text{new} = 120 \\ p = .35 \\ b = \text{unknown} \end{cases}$$

$$120 = b + .35b$$

$$120 = 1.35b$$

$$\frac{120}{1.35} = b$$

$$b = \$88.89$$

Write formula

Identify values

Substitute

Combine like terms

Isolate variable

*** CAUTION *** Always round \$ answers to two decimal places, even when it doesn't say to round.

Percent Decrease also means 2 steps in a single formula

1. Find p percent of b .
2. Subtract result from b .

$$\text{new amount} = b - pb$$

$$\text{or } \text{new amt} = (1-p)b$$

*** SAME CAUTION *** b must be the old amount or the amount that was at an earlier time.

Must not confuse b and the new level or you'll get the wrong answer.

Types of problems that are percent decrease:

- 1) Pay cut
- 2) Investment loss
- 3) Discount pricing (reg. price to sale price)
- 4) Population decrease
- 5) "Percent Decrease"

- ③ After losing 9% of ~~the~~ investment, the account held \$22,750. What was the original investment?

Investment loss $\Rightarrow$ Percent decrease

$$\begin{cases} \text{new amount} = 22750 \\ p = .09 \\ b = \text{unknown} \end{cases}$$

Identify values

m45 SSM 2/e Day 2 p.3
ex(3) cont

$$\text{new amt} = b - p \cdot b$$

$$22750 = b - .09b$$

$$22750 = (1 - .09)b$$

$$22750 = .91b$$

$$\frac{22750}{.91} = b$$

$$b = \$25000$$

Write formula

Substitute

Combine like terms

Isolate variable

Write answer with units

- ④ A suit discounted 30% has a sale price of \$399. What was the original price?

Sale price $\Rightarrow$ percent decrease

$$\left\{ \begin{array}{l} \text{new amt} = 399 \\ p = .3 \\ b = \text{unknown} \end{array} \right.$$

$$\left\{ \begin{array}{l} p = .3 \\ b = \text{unknown} \end{array} \right.$$

$$\text{new amt} = b - .3b$$

$$399 = .7b$$

$$\frac{399}{.7} = b$$

$$b = \$570$$

Write values

Write formula

Substitute

Combine like terms

Isolate variable

Write answer with units.

- Mixed Practice - See Handout

Problems:

- H-1. A real estate commission is 3% of the selling price of \$123,000. How much is the commission?

$$\begin{aligned}\text{Commission} &= (.03)(123,000) \\ &= \boxed{\$3690}\end{aligned}$$

$$a = p \cdot b$$

$$\begin{cases} a = 123000 \\ p = 3\% = .03 \\ b = \text{unknown} \end{cases}$$

Percent Equation

- H-2. Your exam score is a percentage of total points available. You earned 128 points out of 160 points. What percentage did you get?

$$\frac{128}{160} = \boxed{80\%}$$

$$a = p \cdot b$$

$$\begin{cases} a = 128 \\ b = 160 \\ p = \text{unknown} \end{cases}$$

Percent Equation

$$128 = p \cdot 160$$

divide both by 160.

- H-3. You earn a 5% pay raise. Your new hourly rate is \$23.70. What was your previous hourly rate?

$$23.70 = \text{old rate} + (.05) \text{old rate}$$

$$x + .05x = 23.7$$

$$1.05x = 23.7$$

$$x = 22.571428\dots$$

$$x \approx \boxed{\$22.57}$$

$$\text{new} = b + pb$$

$$23.70 = b + .05b$$

Percent Increase

* CAUTION *

If the problem's answer is money, you should round to the nearest hundredth even if the instructions do not say to round.

- H-4. Your boss gives everyone a 10% pay cut. Your previous salary was \$120,000. What is your new salary?

$$\text{new} = \text{old} - .10(\text{old})$$

$$\text{new} = 120,000 - (.10)(120,000)$$

$$= \boxed{\$108,000}$$

$$\text{new} = b - p \cdot b$$

$$\begin{cases} \text{new} = \text{unknown} \\ p = .1 \\ b = 120,000 \end{cases}$$

Percent Decrease

- H-5. A store marks up the wholesale price of all clothes by 35%. If the selling price of a shirt is \$56, what was the wholesale price?

$$\text{Selling price} = \text{wholesale} + .35(\text{wholesale})$$

$$56 = x + .35x$$

$$56 = 1.35x$$

$$x = 41.481481\dots$$

$$x \approx \boxed{\$41.48}$$

$$\text{new} = b + pb$$

$$\begin{cases} \text{new} = 56 \\ p = .35 \\ b = \text{unknown} \end{cases}$$

Percent Increase

Extra Practice

- ⑤ House value increased 4% over last year, now worth \$208,000. What was the value last year?

Percent increase

$$\text{new} = b + pb$$

$$\begin{cases} \text{new} = 208,000 \\ p = .04 \\ b = \text{unknown} \end{cases}$$

$$208,000 = b + .04b$$

$$208,000 = 1.04b$$

$$\frac{208,000}{1.04} = b$$

$$1.04$$

$$\boxed{b = \$200,000}$$

- ⑥ Population of Carlsbad NM decreased from 28,000 to 25,000. Find the % decrease to nearest tenth of a percent.

Percent decrease

$$\text{new} = b - pb$$

$$\begin{cases} \text{new} = 25,000 \\ b = 28,000 \\ p = \text{unknown} \end{cases}$$

Write formula

Identify values

$$25,000 = 28,000 - p(28,000)$$

$$25,000 = 28,000 - 28,000p$$

$$\underline{-28,000} \quad \underline{-28,000}$$

$$-3,000 = -28,000p$$

$$\underline{-3,000} = p$$

$$\underline{-28,000}$$

$$p = \frac{3}{28} = .107142857143$$

$$p \approx .107$$

$$\boxed{p = 10.7\%}$$

Substitute

Isolate p.

Subtract 28,000 both sides

- ⑦ You bought a tractor last year for \$3750 and sell it this year for \$3900. What is the rate of inflation?

inflation $\Rightarrow$ percent increase

$$\text{new} = b + pb$$

$$\begin{cases} \text{new} = 3900 \\ b = 3750 \\ p \text{ unknown} \end{cases}$$

Write formula

Identify values

M45 SSM 2/e 2.6 Day 2 p.6
Extra Practice ⑦ cont

$$\begin{array}{r} 3900 = 3750 + p \cdot 3750 \\ 3900 = 3750 + 3750p \\ -3750 \quad -3750 \end{array}$$

$$150 = 3750p$$

$$\frac{150}{3750} = p$$

$$p = .04$$

$$p = 4\%$$

Substitute

rewrite

subtract 3750

Divide by 3750

Isolate variable

Convert decimal to percent.

- ⑧ A 40% sale is advertised. A coat you've been watching is \$75 originally. What is the sale price?

Sale price $\Rightarrow$ Percent decrease

$$\text{new} = b - pb$$

{ new = unknown

$$b = 75$$

$$p = .4$$

$$x = 75 - .4(75)$$

$$x = \$45$$

Write formula

Find values

Calculate.

BONUS FEATURE

How to solve percent increase and percent decrease problems using a percent proportion.

* CAUTION *

You must memorize the lists of types of problems which are increase or decrease no matter which method you use.

Example ① $\left. \begin{array}{l} \text{new} = 8460 \\ p = 5.75\% \\ b = \text{unknown} \end{array} \right\} \text{percent increase}$

Step 1: Identify values as before, but write p as a percent, (not a decimal).

Step 2: For percent increase add $100\% + p \Rightarrow 100 + 5.75 = 105.75\%$
For percent decrease subtract $100\% - p$

step 3: Write a proportion

$$\frac{\text{new}}{b} = \frac{[\text{result from step 2}]}{100}$$

step 4: Substitute values

$$\frac{8460}{b} = \frac{105.75}{100}$$

step 5: Cross-multiply.

$$(8460)(100) = 105.75 b$$

$$846000 = 105.75 b$$

step 6: Isolate variable using algebra

$$\frac{846000}{105.75} = b$$

$$b = \$8000$$

Note: Either method, used correctly, should get the same answer!

② mark up $\Rightarrow$ percent increase

$$\begin{cases} \text{new} = 120 \\ p = 35\% \\ b = \text{unknown} \end{cases}$$

$$100\% + 35\% = 135\%$$

$$\frac{\text{new}}{b} = \frac{p}{100}$$

$$\frac{120}{x} = \frac{135}{100}$$

$$12000 = 135x$$

$$\frac{12000}{135} = x$$

$$x = \$88.89$$

yes! you could also reduce $\frac{135}{100}$ first!

Identify values.

Add 100%

Write proportion

Substitute

cross-multiply

Round as before.

③ Investment loss $\Rightarrow$ percent decrease

$$\begin{cases} \text{new} = 22750 \\ p = 9\% \\ b = \text{unknown} \end{cases}$$

identify values

$$100\% - 9\% = 91\%$$

subtract from 100% for percent decrease

$$\frac{\text{new}}{b} = \frac{91\%}{100\%}$$

substitute

$$\frac{22750}{x} = \frac{91}{100}$$

$$2275000 = 91x$$

cross multiply

$$\frac{2275000}{91} = x$$

$$x = \$25000$$

④ discount price $\Rightarrow$ percent decrease

$$\begin{cases} \text{new} = 399 \\ p = 30\% \\ b = \text{unknown} \end{cases}$$

$$100\% - 30\% = 70\%$$

$$\frac{\text{new}}{b} = \frac{70\%}{100\%}$$

$$\frac{399}{x} = \frac{70}{100}$$

$$39900 = 70x$$

$$x = \frac{39900}{70}$$

$$x = \$570$$

Note: You can often choose to reduce fractions in percent proportions to get smaller numbers:

$$\frac{399}{x} = \frac{7}{10}$$

$$3990 = 7x$$

$$x = \frac{3990}{7} = \$570$$

- Handout - Pgs proportions

H-① commission $\Rightarrow$ basic percent

$$\frac{a}{b} = \frac{p}{100}$$

write basic % proportion

$$\begin{cases} a = \text{unknown} \\ p = 3\% \\ b = 123,000 \end{cases}$$

Find values

$$\frac{x}{123,000} = \frac{3}{100}$$

substitute

$$100x = 369000$$

$$x = \frac{369000}{100}$$

$$x = \$3690$$

cross-multiply

isolate variable

H-② exam score $\Rightarrow$ basic percent

$$\begin{cases} a = 128 \\ b = 160 \\ p = \text{unknown} \end{cases}$$

$$\frac{a}{b} = \frac{p}{100}$$

$$\frac{128}{160} = \frac{p}{100}$$

$$12800 = 160p$$

$$\frac{12800}{160} = p$$

$$p = 80\%$$

or reduce! $\frac{4}{5} = \frac{p}{100}$

$$400 = 5p$$

$$\frac{400}{5} = p$$

$$p = 80\%$$

Note: When using the percent proportion, the percents go in and come out as %, not decimal

H-③ pay raise $\Rightarrow$ percent increase

$$\begin{cases} \text{new} = 23.70 \\ p = 5\% \\ b = \text{unknown} \end{cases}$$

$$100\% + 5\% = 105\%$$

$$\frac{\text{new}}{b} = \frac{105}{100}$$

$$\frac{23.70}{x} = \frac{105}{100}$$

$$2370 = 105x$$

$$x = \frac{2370}{105}$$

$$x = \$22.57$$

or reduce! $\frac{23.70}{x} = \frac{21}{20}$

$$474 = 21x$$

$$x = \frac{474}{21}$$

$$x = \$22.57$$

H-④ pay cut $\Rightarrow$ percent decrease

$$\begin{cases} \text{new} = \text{unknown} \\ p = 10\% \\ b = 120,000 \\ 100\% - 10\% = 90\% \end{cases}$$

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handout ④ cont

Bonus - Proportions

$$\frac{\text{new}}{b} = \frac{90}{100}$$

$$\frac{x}{120,000} = \frac{90}{100}$$

$$100x = 10800000$$

$$x = \frac{10800000}{100}$$

$$x = \$108,000$$

or reduce!

$$\frac{x}{120,000} = \frac{9}{10}$$

$$10x = 1080000$$

$$x = \frac{1080000}{10}$$

$$x = \$108,000$$

H- ⑤ mark up $\Rightarrow$ percent increase

$$\begin{cases} \text{new} = 56 \\ p = 35\% \\ b = \text{unknown} \end{cases}$$

$$100\% + 35\% = 135\%$$

$$\frac{\text{new}}{b} = \frac{135}{100}$$

$$\frac{56}{x} = \frac{135}{100}$$

$$5600 = 135x$$

$$x = \frac{5600}{135}$$

$$x = \$41.48$$

or reduce!

$$\frac{56}{x} = \frac{27}{20}$$

$$1120 = 27x$$

$$x = \frac{1120}{27}$$

$$x = \$41.48$$

Extra Practice - by proportions

⑤ percent increase

$$\begin{cases} \text{new} = 208,000 \\ p = 4\% \\ b = \text{unknown} \end{cases}$$

$$100\% + 4\% = 104\%$$

$$\frac{\text{new}}{b} = \frac{104}{100}$$

$$\frac{208000}{x} = \frac{104}{100}$$

$$20800000 = 104x$$

$$x = \frac{20800000}{104}$$

$$x = \$200,000$$

or reduce!

$$\frac{208000}{x} = \frac{26}{25}$$

$$5200000 = 26x$$

$$x = \frac{5200000}{26}$$

$$x = \$200,000$$

⑥ percent decrease

$$\begin{cases} \text{new} = 25000 \\ p = \text{unknown} \\ b = 28000 \end{cases}$$

$$100\% - x = 100 - x$$

← uh-oh! Need an expression!

$$\frac{\text{new}}{b} = \frac{100 - x}{100}$$

← Substitute the expression

$$\frac{25000}{28000} = \frac{100 - x}{100}$$

cross multiply

$$25000x = (100 - x)28000$$

$$25000x = 2800000 - 28000x$$

distribute

$$+ 28000x$$

$$+ 28000x$$

isolate variable
→ collect x first

$$53000x = 2800000$$

$$x = \frac{2800000}{53000}$$

$$x = 10.7\%$$

⑦ inflation ⇒ percent increase

$$\begin{cases} \text{new} = 3900 \\ p = \text{unknown} \\ b = 3750 \end{cases}$$

$$100 + x$$

← Need an expression!

$$\frac{\text{new}}{b} = \frac{100 + x}{100}$$

$$\frac{3900}{3750} = \frac{100 + x}{100}$$

$$390000 = 3750(100 + x)$$

$$\begin{array}{r} 390000 = 375000 + 3750x \\ -375000 \quad -375000 \\ \hline 15000 = 3750x \end{array}$$

$$15000 = 3750x$$

$$x = 4\%$$

⑧ sale price ⇒ percent decrease

$$\text{new} = \text{unknown}$$

$$b = 75$$

$$p = 40\%$$

$$100\% - 40\% = 60\%$$

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⑧ cont

$$\frac{\text{new}}{b} = \frac{60\%}{100\%}$$

$$\frac{x}{75} = \frac{60}{100}$$

$$100x = 4500$$

$$x = \frac{4500}{100}$$

$$\boxed{x = \$45}$$

or reduce!

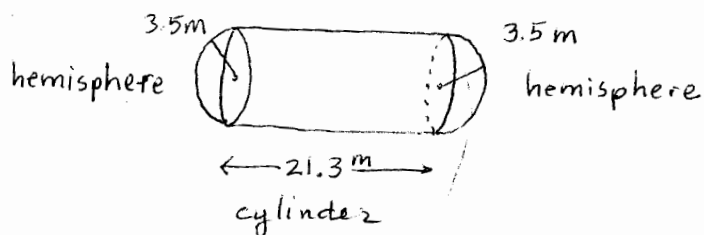
$$\frac{x}{75} = \frac{3}{5}$$

$$5x = 225$$

$$x = \frac{225}{5}$$

$$\boxed{x = \$45}$$

yes (6) Volume of a tank



volume =
always
cubic units

a) Find the exact value of the volume of the tank.

step 1: volume of cylinder $V = \pi r^2 h$ $r = 3.5 \text{ m}$
 $h = 21.3 \text{ m}$

$$V = \pi (3.5)^2 (21.3)$$

* Exact * means "in terms of π " $\Rightarrow$ so π is a symbol in answer

$$V = 260.925 \pi \text{ m}^3$$

step 2: volume of sphere $V = \frac{4}{3} \pi r^3$ $r = 3.5 \text{ m}$

* Again, exact, in terms of π :

$$V = \frac{4}{3} \cdot (3.5)^3 \cdot \pi$$

use >frac to get exact answer
w/o rounding or repeat bars

$$V = \frac{343}{6} \pi \text{ m}^3$$

step 3: Add exact results, combine like terms, use >frac

$$V = 260.925 \pi + \frac{343}{6} \pi$$

$$V = (260.925 + \frac{343}{6}) \pi$$

$$V = \frac{38171}{120} \pi \text{ m}^3 \quad \text{exact answer}$$

b) Find the approximate value to nearest hundredth.

step 1: Calculate exact result, using π button on GC

$$38171 / 120 * \pi \approx 999.3144432$$

step 2: Round to nearest hundredth

$$\boxed{999.31 \text{ m}^3} \quad \text{approximate answer}$$

c) MyMathLab or MathXL sometimes requires you to use a specific rounded value of π .

"Find the approximate volume of the tank to nearest hundredth.
Use $\pi \approx 3.1416$."

step 1: Calculate exact result using value of π they give:

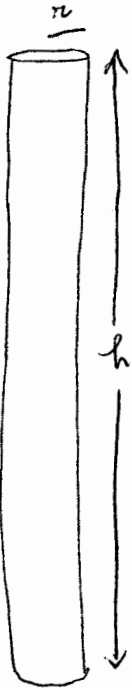
$$38171 / 120 * 3.1416 \approx 999.31678$$

step 2: Round to nearest hundredth: $\boxed{999.32 \text{ m}^3}$

7

Scientists are drilling a hole in the ocean floor to learn more about the Earth's history. Currently, the hole is in the shape of a cylinder whose volume is approximately 3700 cubic feet and whose height is 1.1 miles. Find the radius of the hole to the nearest hundredth of a foot. (Hint: Make sure the same units of measurement are used.)

The radius of the hole is 0.45 feet. (Round to the nearest hundredth.)



Volume of a cylinder $V = \pi r^2 h$

$$V = 3700 \text{ ft}^3$$

$$h = 1.1 \text{ miles}$$

units are different!

Either: convert all to feet or all to miles.

$$1.1 \text{ mi} \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) = 5808 \text{ ft.}$$

$$h = 5808 \text{ ft}$$

$$V = \pi r^2 h$$

$$3700 = \pi r^2 (5808)$$

$$\frac{3700}{\pi \cdot 5808} = r^2$$

$$+ \sqrt{\frac{3700}{\pi \cdot 5808}} = r$$

$$.4503 = r$$

$$r \approx .45 \text{ feet}$$

Exact answer.

The negative sq. root is extraneous.

Notice that the answer box is given in feet, so convert miles to feet.